Yugoslav IMO Team Selection Test 1995

Vrbas, April 16, 1995.

Time allowed 180 *minutes.*
Each problem is worth 25 *points.*

1. Determine all triples (x, y, z) of positive rational numbers with $x \le y \le z$ such that $x + y + z$, $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ and xyz are natural numbers.

2. A natural number n has exactly 1995 units in its binary representation. Show that $n!$ is divisible by 2^{n-1995}.

3. Let $SABCD$ be a pyramid with the vertex S whose all edges are equal. Points M and N on the edges SA and BC respectively are such that MN is perpendicular to both SA and BC. Find the ratios $SM : MA$ and $BN : NC$.