

36-th International Mathematical Olympiad

Toronto, Canada, July 13–25, 1995

First Day – July 19

1. Let A, B, C , and D be distinct points on a line, in that order. The circles with diameters AC and BD intersect at X and Y . O is an arbitrary point on the line XY but not on AD . CO intersects the circle with diameter AC again at M , and BO intersects the other circle again at N . Prove that the lines AM , DN , and XY are concurrent. *(Bulgaria)*

2. Let a, b , and c be positive real numbers such that $abc = 1$. Prove that

$$\frac{1}{a^3(b+c)} + \frac{1}{b^3(a+c)} + \frac{1}{c^3(a+b)} \geq \frac{3}{2}. \quad \text{(Russia)}$$

3. Determine all integers $n > 3$ such that there are n points A_1, A_2, \dots, A_n in the plane that satisfy the following two conditions simultaneously:

- (a) No three lie on the same line.
(b) There exist real numbers p_1, p_2, \dots, p_n such that the area of $\triangle A_i A_j A_k$ is equal to $p_i + p_j + p_k$, for $1 \leq i < j < k \leq n$. *(Czech Republic)*

Second Day – July 20

4. The positive real numbers $x_0, x_1, \dots, x_{1995}$ satisfy $x_0 = x_{1995}$ and

$$x_{i-1} + \frac{2}{x_{i-1}} = 2x_i + \frac{1}{x_i}$$

for $i = 1, 2, \dots, 1995$. Find the maximum value that x_0 can have.

(Poland)

5. Let $ABCDEF$ be a convex hexagon with $AB = BC = CD$, $DE = EF = FA$, and $\angle BCD = \angle EFA = \pi/3$ (that is, 60°). Let G and H be two points interior to the hexagon, such that angles AGB and DHE are both $2\pi/3$ (that is, 120°). Prove that $AG + GB + GH + DH + HE \geq CF$.

(New Zealand)

6. Let p be an odd prime. Find the number of p -element subsets A of $\{1, 2, \dots, 2p\}$ such that the sum of all elements of A is divisible by p .

(Poland)