

17-th Balkan Mathematical Olympiad

Chişinău, Moldova – May 5, 2000

1. [BMO 1997#4] Determine all functions $f : \mathbb{R} \rightarrow \mathbb{R}$ that satisfy

$$f(xf(x) + f(y)) = f(x)^2 + y \quad \text{for all } x, y. \quad (\text{Albania})$$

2. Let ABC be a scalene triangle and E be a point on the median AD . Point F is the orthogonal projection of E onto BC . Let M be a point on the segment EF , and N, P be the orthogonal projections of M onto AC and AB respectively. Prove that the bisectors of the angles PMN and PEN are parallel.
3. Find the maximal number of rectangles $1 \times 10\sqrt{2}$ that can be cut off from a rectangle 50×90 by using cuts parallel to the edges of the big rectangle.
(Yugoslavia)
4. A positive integer is a *power* if it is of the form t^s for some integers $t, s \geq 2$. Prove that for any natural number n there exists a set A of positive integers with the following properties:
- (i) A has n elements;
 - (ii) Every element of A is a power;
 - (iii) For any $2 \leq k \leq n$ and any $r_1, \dots, r_k \in A$, $\frac{r_1 + \dots + r_k}{k}$ is a power.