42-nd International Mathematical Olympiad

Washington DC, United States of America, July 1–14, 2001

First Day – July 8

1. In acute triangle ABC with circumcenter O and altitude AP, $\measuredangle C \geq \measuredangle B + 30^\circ$. Prove that $\measuredangle A + \measuredangle COP < 90^\circ$. *(South Korea)*

2. Prove that for all positive real numbers a, b, c,
$$\frac{a}{\sqrt{a^2+8bc}} + \frac{a}{\sqrt{b^2+8ca}} + \frac{c}{\sqrt{c^2+8ab}} \geq 1.$$ *(South Korea)*

3. Twenty-one girls and twenty-one boys took part in a mathematical competition. It turned out that

   (i) each contestant solved at most six problems, and

   (ii) for each pair of a girl and a boy, there was at least one problem that was solved by both the girl and the boy.

   Show that there is a problem that was solved by at least three girls and at least three boys. *(Germany)*

Second Day – July 9

4. Let n be an odd integer greater than 1 and let $c_1, c_2, \dots, c_n$ be integers. For each permutation $a = (a_1, a_2, \dots, a_n)$ of $\{1, 2, \dots, n\}$, define $S(a) = \sum_{i=1}^{n} c_i a_i$. Prove that there exist permutations $a \neq b$ of $\{1, 2, \dots, n\}$ such that $n!$ is a divisor of $S(a) - S(b)$. *(Canada)*

5. Let ABC be a triangle with $\measuredangle BAC = 60^\circ$. Let AP bisect $\angle BAC$ and let BQ bisect $\angle ABC$, with P on BC and Q on AC. If $AB + BP = AQ + QB$, what are the angles of the triangle? *(Israel)*

6. Let $a > b > c > d$ be positive integers and suppose
$$ac + bd = (b + d + a - c)(b + d - a + c).$$
   Prove that $ab + cd$ is not prime. *(Bulgaria)*