

Serbia and Montenegro Team Selection Test 2003

Novi Sad, April 20, 2003

Time allowed 3 hours.

Each problem is worth 25 points.

1. For a polynomial $p(x)$, denote $p^n(x) = \underbrace{p(p(\cdots p(x)\cdots))}_n$. Prove that the

polynomial

$$p^{2003}(x) - 2p^{2002}(x) + p^{2001}(x)$$

is divisible by $p(x) - x$.

2. Each edge and each diagonal of a convex n -gon ($n \geq 3$) is colored in red or blue. Prove that the vertices of the n -gon can be labelled with A_1, A_2, \dots, A_n so that one of the following two conditions is satisfied:

1° the segments $A_1A_2, A_2A_3, \dots, A_{n-1}A_n, A_nA_1$ are of the same color;

2° there exists k , $1 < k < n$, such that the segments $A_1A_2, A_2A_3, \dots, A_{k-1}A_k$ are blue, and the segments $A_kA_{k+1}, \dots, A_{n-1}A_n, A_nA_1$ are red.

3. Suppose M and N are distinct points in the plane of triangle ABC such that

$$AM : BM : CM = AN : BN : CN.$$

Prove that the line MN contains the circumcenter of $\triangle ABC$.