

# Serbia and Montenegro Team Selection Test 2003

Novi Sad, April 20, 2003

*Time allowed 3 hours.*

*Each problem is worth 25 points.*

1. For a polynomial  $p(x)$ , denote  $p^n(x) = \underbrace{p(p(\cdots p(x)\cdots))}_n$ . Prove that the

polynomial

$$p^{2003}(x) - 2p^{2002}(x) + p^{2001}(x)$$

is divisible by  $p(x) - x$ .

2. Each edge and each diagonal of a convex  $n$ -gon ( $n \geq 3$ ) is colored in red or blue. Prove that the vertices of the  $n$ -gon can be labelled with  $A_1, A_2, \dots, A_n$  so that one of the following two conditions is satisfied:

1° the segments  $A_1A_2, A_2A_3, \dots, A_{n-1}A_n, A_nA_1$  are of the same color;

2° there exists  $k$ ,  $1 < k < n$ , such that the segments  $A_1A_2, A_2A_3, \dots, A_{k-1}A_k$  are blue, and the segments  $A_kA_{k+1}, \dots, A_{n-1}A_n, A_nA_1$  are red.

3. Suppose  $M$  and  $N$  are distinct points in the plane of triangle  $ABC$  such that

$$AM : BM : CM = AN : BN : CN.$$

Prove that the line  $MN$  contains the circumcenter of  $\triangle ABC$ .