

45-th International Mathematical Olympiad

Athens, Greece – July 9–19, 2004

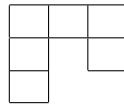
First Day – July 12

1. Let ABC be an acute-angled triangle with $AB \neq AC$. The circle with diameter BC intersects the sides AB and AC at M and N , respectively. Denote by O the midpoint of BC . The bisectors of the angles BAC and MON intersect at R . Prove that the circumcircles of the triangles BMR and CNR have a common point lying on the line segment BC . (*Romania*)
2. Find all polynomials $P(x)$ with real coefficients which satisfy the equality

$$P(a - b) + P(b - c) + P(c - a) = 2P(a + b + c)$$

for all triples a, b, c of real numbers such that $ab + bc + ca = 0$. (*S. Korea*)

3. Determine all $m \times n$ rectangles that can be covered with *hooks* made up of 6 unit squares, as in the figure:



Rotations and reflections of hooks are allowed. The rectangle must be covered without gaps and overlaps. No part of a hook may cover area outside the rectangle. (*Estonia*)

Second Day – July 13

4. Let $n \geq 3$ be an integer and t_1, t_2, \dots, t_n positive real numbers such that

$$n^2 + 1 > (t_1 + t_2 + \dots + t_n) \left(\frac{1}{t_1} + \frac{1}{t_2} + \dots + \frac{1}{t_n} \right).$$

Show that t_i, t_j, t_k are the side lengths of a triangle for all i, j, k with $1 \leq i < j < k \leq n$. (*S. Korea*)

5. In a convex quadrilateral $ABCD$ the diagonal BD does not bisect the angles ABC and CDA . The point P lies inside $ABCD$ and satisfies

$$\angle PBC = \angle DBA \quad \text{and} \quad \angle PDC = \angle BDA.$$

Prove that $ABCD$ is a cyclic quadrilateral if and only if $AP = CP$.

(*Poland*)

6. We call a positive integer *alternative* if its decimal digits are alternatively odd and even. Find all positive integers n such that n has an alternative multiple. (*Iran*)