45-th International Mathematical Olympiad

Athens, Greece – July 9–19, 2004

First Day – July 12

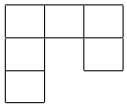
1. Let ABC be an acute-angled triangle with $AB \neq AC$. The circle with diameter BC intersects the sides AB and AC at M and N, respectively. Denote by O the midpoint of BC. The bisectors of the angles BAC and MON intersect at R. Prove that the circumcircles of the triangles BMR and CNR have a common point lying on the line segment BC. *(Romania)*

2. Find all polynomials $P(x)$ with real coefficients which satisfy the equality

$$P(a-b)+P(b-c)+P(c-a)=2P(a+b+c)$$

for all triples a, b, c of real numbers such that $ab+bc+ca=0$. *(S. Korea)*

3. Determine all $m \times n$ rectangles that can be covered with *hooks* made up of 6 unit squares, as in the figure:



Rotations and reflections of hooks are allowed. The rectangle must be covered without gaps and overlaps. No part of a hook may cover area outside the rectangle. *(Estonia)*

Second Day – July 13

4. Let $n \geq 3$ be an integer and $t_1, t_2, \ldots, t_n$ positive real numbers such that

$$n^2+1 > (t_1+t_2+\cdots+t_n)\left(\frac{1}{t_1}+\frac{1}{t_2}+\cdots+\frac{1}{t_n}\right).$$

Show that t_i, t_j, t_k are the side lengths of a triangle for all i, j, k with $1 \leq i < j < k \leq n$. *(S. Korea)*

5. In a convex quadrilateral $ABCD$ the diagonal BD does not bisect the angles ABC and CDA. The point P lies inside $ABCD$ and satisfies

$$\angle PBC = \angle DBA \quad \text{and} \quad \angle PDC = \angle BDA.$$

Prove that $ABCD$ is a cyclic quadrilateral if and only if $AP = CP$. *(Poland)*

6. We call a positive integer *alternative* if its decimal digits are alternatively odd and even. Find all positive integers n such that n has an alternative multiple. *(Iran)*