

27-th Balkan Mathematical Olympiad

Chişinău, Moldova – May 4, 2010

1. If a, b and c are positive real numbers, prove that

$$\frac{a^2b(b-c)}{a+b} + \frac{b^2c(c-a)}{b+c} + \frac{c^2a(a-b)}{c+a} \geq 0 \quad (\text{Saudi Arabia})$$

2. Let ABC be an acute-angled with orthocenter H and let M be the midpoint of AC . The foot of the altitude from C is C_1 . Point H_1 is symmetric to H in AB . The projections of C_1 on lines AH_1, AC and BC are P, Q and R respectively. If M_1 is the circumcenter of triangle PQR , prove that the point symmetric to M with respect to M_1 lies on line BH_1 . (*Serbia*)
3. We define a w -strip as the set of all points in the plane that are between or on two parallel lines on a mutual distance w . Let S be a set of n points in the plane such that any three points from S can be covered by a 1-strip. Show that the entire set S can be covered by a 2-strip. (*Romania*)
4. For every integer $n \geq 2$, denote by $f(n)$ the sum of positive integers not exceeding n that are coprime to n . Prove that $f(n+p) \neq f(n)$ for any such n and any prime number p . (*Turkey*)

Each problem is worth 10 points.

Time allowed: $4\frac{1}{2}$ hours.